# NUMERICALS:COUNTING, MEASURING AND CLASSIFYING 

SUSAN Rothstein<br>Bar-Ilan University

## 1 Basic properties of number words

In this paper, I discuss three different semantic uses of numerical expressions. In their first use, numerical expressions are numerals, or names for numbers. They occur in direct counting situations (one, two, three...) and in mathematical statements such as (1a). Numericals have a second predicative interpretation as numerical or cardinal adjectives, as in (1b). Some numericals have a third use as numerical classifiers as in (1c):
(1) a. Six is bigger than two.
b. Three girls, four boys, six cats.
c. Hundreds of people gathered in the square.

In part one, I review the two basic uses of numericals, as numerals and adjectives. Part two summarizes results from Rothstein 2009, which show that numericals are also used as numerals in measure constructions such as two kilos of flour. Part three discusses numerical classifiers. In parts two and three, we bring data from Modern Hebrew which support the syntactic structures and compositional analyses proposed. Finally we distinguish three varieties of pseudopartitive constructions, each with different interpretations of the numerical: In measure pseudopartitives such as three kilos of books, three is a numeral, in individuating pseudopartitives such as three boxes of books, three is a numerical adjective, and in numerical pseudopartitives such as hundreds of books, hundreds is a numerical classifier.

### 1.1 Basic meanings for number word

### 1.1.1 Number words are names for numbers

Numericals occur bare as numerals in direct counting contexts in which we count objects (one, two, three) and answer questions such as how many $N$ are there? and in statements such as (1a)
and (2). They name numbers and semantically are analogous to proper names which name individuals:
(2) a. Two, four, six and eight are the first four even numbers.
b. Two is the only even prime number.
c. Two times two is four. / Two plus two is four.

I assume that numbers are abstract entities and numerals are proper names for numbers at type n . (1a) and (2) shows that numbers, like other individuals, have properties. (2c) shows that plus, and times denote operations on numbers and are of type $<\mathrm{n},<\mathrm{n}, \mathrm{n} \gg$ as illustrated in (3).
(3) times: $\lambda \mathrm{n} \lambda \mathrm{n} . \mathrm{n} \times \mathrm{n}$
times two: $\lambda \mathrm{n} . \mathrm{n} \times 2$
two times two: $2 \times 2$
The singular agreement in the verb in (2c), as opposed to the plural agreement in (2a) is an indication of the fact that two times two is a complex numeral, denoting the number 4. Numerals can also be conjoined with and as in ( $4 \mathrm{a}-\mathrm{c}$ ). In this case, plural agreement on the verb is common in English, although this is not an absolute requirement. In other languages (e.g. Dutch) agreement must be singular (4d). This means that and in English is not normally an expression of type $<\mathrm{n},<\mathrm{n}, \mathrm{n} \gg$, but denotes a function from pairs of numbers into pluralities.
(4) a. "Two and two make five." (Orwell, 1984: Part III, chapter 4)
b. "Two and two are four, four and four are eight..."(Danny Kay, Inchworm)
c. "Now one and one is two mama, two and two is four," (Robert Johnson:

Sweet Home Chigago)
d. Twee en twee is vier. (Dutch)

Two and two is-SG four.
Numericals also have a nominal interpretation at the predicate type and behave like normal nouns. In (5a), the N predicate four combines with a determiner, and in (5b) twos is pluralized and modified by a cardinal adjective. In this too, they look like proper names, see (6):
(5) a. Move the four on the right of the equation to the left hand-side.
b. Two twos are four, three twos are six.
c. "What are twelve sevens?" (Roald Dahl, Matilda)
(6) There are two Johns in the class. The John I am talking about sits on the right.

Numericals like hundred, thousand, million must combine with another numerical in order to form a numeral, as shown in (7):
(7) a. Two/one hundred people stood in line.
b.*Hundred people stood in line.

Note that there is a contrast between two hundred which is a numeral, and two hundreds, analogous to two twos in (5b), which counts instances of one hundred. This shows up in (8):
(8) a. Nine hundreds are nine hundred, ten hundreds are a thousand.
b. Nine hundred is nine hundred

While both the examples in (8) are tautological, (8a), with plural agreement on the verb, is informative in the same way that The Morning Star is the Evening Star is informative, while (8b), where the verb has singular agreement, is uninformative, analogous to The Morning Star is the Morning Star.

There is thus good evidence that numerals are grammatically names for individual numbers, and are analogous to proper names which denote singular individuals. I assume that a number $n$ (i.e. the denotation of a numeral) is an equivalence class of entities with cardinality 2 , i.e. entities constructed out of two atomic parts: $2=\{x:|x|=2\}$. A numeral $n$ has the denotation in (9a), ${ }^{1}$ and thus a simple numeral, such as $t w o$, has the interpretation in ( 9 b ). The corresponding predicate nominal interpretation at type $<\mathrm{d}, \mathrm{t}>$ is the set $\{\mathrm{x}:|\mathrm{x}|=2\}$. There are also complex numerical expressions such as hundred, which are of type $<\mathrm{n}, \mathrm{n}>$ and combine with a numeral to form a complex numeral denoting the number $\mathrm{n} \times 100$, as in (9c). We will call these 'multiplicative numerals', and discuss them further in section 3.
a. $\mathrm{n}=\{\mathrm{x}:|\mathrm{x}|=\mathrm{n}\}=\lambda \mathrm{x} .\left|\left\{\mathrm{y}: \mathrm{y} \sqsubseteq_{\text {Атом }} \mathrm{x}\right\}\right|=\mathrm{n}$

The numeral $n$ denotes the set of entities with $n$ non-overlapping atomic parts.
b. $\llbracket$ two $\rrbracket=\{\mathrm{x}:|\mathrm{x}|=2\}=\lambda \mathrm{x} .\left|\left\{\mathrm{y}: \mathrm{y} \sqsubseteq_{\text {Атом }} \mathrm{x}\right\}\right|=2$
c. $\llbracket$ hundred $\rrbracket=\lambda \mathrm{n} .\{\mathrm{x}:|\mathrm{x}|=100 \times \mathrm{n}\}$

### 1.1.2 Numericals as cardinal adjectives

Numericals have a predicative use as cardinal adjectives, exploiting the meaning in (9):
(10) a. The guests are two.
b. The two guests arrived.
c. Two guests arrived.
d. The musicians are two of our friends.
e. Two of the guests arrived.

Following Landman 2003, we assume that in (10c), the numerical is an adjective which raises to determiner position if there is no determiner. The adjective two has a predicate denotation derived from (9), and given in (11a): it denotes the property an entity has if the cardinality of the set of its atomic part is 2 . Raising to determiner position induces type-shifting to the type of generalized quantifiers, as in (11b):
a. $t w o_{<d, 1>}: \lambda x .|x|=2$

$$
\begin{equation*}
=\lambda \mathrm{x} .\left|\left\{\mathrm{y}: \mathrm{y} \sqsubseteq_{\text {АTOM }} \mathrm{x}\right\}\right|=2 \tag{11}
\end{equation*}
$$

b. $t w o_{\ll d, t>, \ll d, t>, t>}: \lambda Q \lambda P . \exists \mathrm{x}[\mathrm{P}(\mathrm{x}) \wedge \mathrm{Q}(\mathrm{x}) \wedge|\mathrm{x}|=2]$

The meanings in (11) are used in the interpretations of (10-c) as follows:

[^0](12) a. The guests are two:
$|\sigma\{\mathrm{x}: \operatorname{GUESTS}(\mathrm{x})\}|=2$
"The cardinality of the unique maximal sum of the set of guests is two."
$=$ "Being two is a property that the maximal sum of guests has."
b. The two guests arrived:
$\operatorname{ARRIVED}(\sigma\{\mathrm{x}: \operatorname{GUESTS}(\mathrm{x}) \wedge|\mathrm{x}|=2\})$
"The maximal sum of guests, whose cardinality is two, arrived".
c. Two guests arrived
$\exists x[\operatorname{GUESTS}(x) \wedge \operatorname{ARRIVED}(\mathrm{x}) \wedge|x|=2]$
"There is a plural individual in GUESTS with two atomic parts who arrived."
Rothstein 2010a shows that a numerical in a partitive construction such as (10d,e) has its standard adjectival interpretation. Two of the guests denotes the set of parts of the plural entity denoted by the guests which have two atomic parts. See Rothstein 2010a for details.

## 2 Number interpretation in pseudopartitives:

### 2.1 Counting vs. measuring

The main use of a cardinal adjective is to modify a count (but not a mass) noun:
(13) three flowers/four books/*three flour(s) .

However, cardinal adjectives also modify classifiers as in (14):
(14) a. Container classifiers: two cups of flour/two cups of beans/two glasses of water
b. Measure classifiers: two kilos of flour/two kilos of beans/two glasses of water

Selkirk 1977, Doetjes 1997, Chierchia 1998, Landman 2004, Rothstein 2009 and others have all showed that classifier phrases like two glasses of water are ambiguous between an 'individuating' reading, illustrated in (15a) and a measure reading in (15b):
(15) a. Mary, bring two glasses of water for our guests!
b. Add two glasses of water to the soup!

Despite the surface similarity of these pseudopartitive constructions, Rothstein 2009, 2010b shows that different grammatical constructions are associated with each reading, and that the numerical is interpreted differently in each case. In (15a) two glasses of water is interpreted with its individuating or container reading, and denotes pluralities of glasses whose cardinality is 2 which contain water. On this reading, two is a predicate expression, giving the cardinality property of the plurality of glasses. (15b) illustrates the measure reading. Here two glasses of water denotes quantities of water, whose measure on the scale of volume, calibrated in terms of glass-measures, is two. Two is a numeral interpreted at type n, and denotes a number, indicating a value on the scale.

Rothstein 2009 shows that these readings can be disambiguated by a variety of tests. For example, on the measure readings, a classifier expression can be marked with $-f u l$ and on an individuating reading this is not possible (16). Distributive expressions such as each distribute
over individuals in the denotation of individuating classifier expressions and are unacceptable with measure phrases (17):
(16) a. Bring two glasses(\#ful) of wine for our guests! (individuating/container reading)
b. Add two glasses(ful) of wine to the soup! (measure reading)
(17) a. Two packs of flour cost 2 euros each.
b. \#Two kilos of flour cost 2 euros each.
c. The two glasses of wine (\#in this soup) cost 2 Euros each.

Container-classifiers are ambiguous between a measure and an individuating interpretation, as shown in (15). Classifier constructions with explicit measure heads such as kilo and litre are most naturally given a measure interpretation, but can be coerced into the individuating reading. On the individuating reading, twenty litres of soda is reinterpreted as twenty litre-containers of soda. (Dutch marks this reinterpretation morphologically: twintig liter frisdrank denotes softdrinks to the measure of 20 litres, while twintig liters frisdrank, with plural morphology on the measure expression liters, denotes twenty litre-containers of soft-drink. See Doetjes 1997).

Measure and individuating pseudopartitives reflect two different ways of assigning numerical values to entities. Counting is putting atomic entities in one-to-one correlation with the natural numbers. The counting operation presupposes a set of atomic entities, and Rothstein 2010a argues that this presupposition is grammatically encoded in the meaning of count nouns, which for this reason denote sets of countable entities. Measuring is giving a value to a quantity on a calibrated dimensional scale, as in ten kilos of flour/books.

While count nouns are directly countable, classifier constructions, or pseudopartitives, are used to express measuring and indirect counting. Container classifiers, such as three boxes of $N$ allow indirect counting of pluralities and mass entities by repackaging the denotations of plural count nouns and mass nouns into atomic containers which can then be counted. We call this the 'individuating' or 'counting' interpretation of classifiers, since we individuate higher-order entities such as boxes and count them. Thus in container classifier constructions, the numerical is interpreted as a cardinal adjective at type <d,t>, as in ordinary count noun modification, and modifies the nominal denoting the container. Measure classifiers such as three kilos of $N$, three litres/bottles of $N$ do not count, but give measure properties. A measure property is the property of having a particular measure value on a dimensional scale. A measure value is an ordered pair $<\mathrm{n}, \mathrm{U}>$, consisting of a number $n$ and a unit, U , the unit in terms of which the dimensional scale is calibrated. In measure constructions, the numerical is a numeral interpreted as type $n$, and it combines with a measure head at type <n,<d,t>> to form a measure predicate. Rothstein 2009, following Landman 2004, shows that individuating and measure interpretations have different compositional interpretations: (18) gives the structure of three glasses of water on the individuating reading. Glasses is the nominal head, and two is an adjective giving the cardinality of the relevant pluralities of glasses.


In order to give the interpretation, we first define the REL $_{\text {contain }}$ operation which shifts the nominal glasses from a set of individual to a relational nominal. REL $_{\text {contain }}$ uses the CONTAIN relation ${ }^{2}$. The CONTAIN relation is given in (19a), and REL $_{\text {contain }}$ in (19b).
a. $\forall \mathrm{x}, \mathrm{y}: \operatorname{CONTAIN}(\mathrm{x}, \mathrm{y}) \rightarrow \forall \mathrm{z}: \mathrm{z} \sqsubseteq_{\text {Атом }} \mathrm{x}: \exists \mathrm{y}^{\prime}: \mathrm{y}^{\prime} \sqsubseteq \mathrm{y}: \operatorname{CONTAIN}(\mathrm{z}, \mathrm{y}$ '). "x CONTAINs $y$ if the atomic parts of $x$ CONTAIN parts of $y$ "
b. $\operatorname{REL}_{\text {contain }}(\lambda x . P(x))=\lambda y \lambda x . P(x) \wedge \operatorname{CONTAIN}(x, y)$

The interpretation of (18) is given in (20). Note that for simplicity, we treat the bare noun in DP position as a kind-denoting term (Carlson 1977, Chierchia 1998).
(20) $\llbracket$ glasses $\rrbracket=$ GLASSES $=P L(G L A S S)=\{x: \exists Y: Y \subseteq G L A S S: ~ x=\sqcup Y\}$
$\operatorname{REL}_{\text {contain }}(\llbracket$ glasses $\rrbracket)=\lambda y \lambda x . x \in \operatorname{GLASSES} \wedge \operatorname{CONTAIN}(\mathrm{x}, \mathrm{y})$
【glasses of wine】 $=\lambda x . x \in$ GLASSES $\wedge$ CONTAIN( $x$, WINE)
$\llbracket$ three glasses of wine $=\lambda x . x \in \operatorname{GLASSES} \wedge \operatorname{CONTAIN}(\mathrm{x}, \mathrm{WINE}) \wedge|\mathrm{x}|=3$
Measure pseudopartitives have the structure in (21), and the interpretation in (22)/(23):
(21)


[^1]The measure head litre is interpreted at type $<\mathrm{n},<\mathrm{d}, \mathrm{t} \gg$, and combines with the numeral at type n , to give three litres, which expresses the measure property $\lambda \mathrm{x} . \operatorname{MEAS}_{\text {volume }}(\mathrm{x})=<3$, LITRE $>$.
(22) three litres of water:
litre: $\quad \lambda \mathrm{n} \lambda \mathrm{x} . \operatorname{MEAS}_{\text {VOLUME }}(\mathrm{x})=<\mathrm{n}$, LITRE $>$
three litres: $\lambda \mathrm{x} . \operatorname{MEAS}_{\text {volume }}(\mathrm{x})=<3$, LITRE $>$
three litres of water: $\left.\lambda \mathrm{x} . \operatorname{WATER}(\mathrm{x}) \wedge \operatorname{MEAS} \mathrm{MOLUME}^{\mathrm{V}} \mathrm{x}\right)=<3$, LITRE $>$
Three litres of water denotes the set of quantities of water which measure three litres. Crucially, the interpretation makes no reference to individuable litre units of water, and this is as it should be. The NP denotes a set of quantities of water whose overall measure value on the volume scale is 3 litres. (22) extends naturally to measure interpretations of container classifiers such as three glasses of water. Glass is assigned the same type as explicit measure phrases such as kilo or litre, i.e. at type <n, <d,t>>. It combines first with the numeral three, interpreted at type n. The complex modifier three glasses then applies to the nominal head water. Plural morphology on glasses is morphological agreement and does not reflect the operation of semantic pluralization which operates in the nominal domain. As in (18), of -insertion is a late phenomenon satisfying surface constraints, and of is not semantically interpreted.
(23) three glasses of water:
glass: $\quad \lambda \mathrm{n} \lambda \mathrm{x} . \operatorname{MEAS}_{\text {VOLUME }}(\mathrm{x})=<\mathrm{n}$, GLASS $>$
three glasses: $\lambda \mathrm{x} \cdot \operatorname{MEAS} \mathrm{VOLUME}(\mathrm{x})=<3$, GLASS $>$ three glasses of water: $\lambda \mathrm{x} . \operatorname{WATER}(\mathrm{x}) \wedge \operatorname{MEAS}_{\text {Volume }}(\mathrm{x})=<3$, GLASS $>$

### 2.2 Syntactic support for this analysis

Rothstein 2009 shows that in individuating partitives as in (18), the classifier and the complement form an NP constituent and the numerical expression modifies this constituent. In contrasts, in the measure constructions in (21), the numerical and measure classifier form a constituent. This predicts that in the individuation constructions, an adjective modifying the NP should be possible between the numerical and the NP, while this should be impossible in the measure construction. This prediction is correct, as shown in (24):
(24) a. The waiter brought three expensive glasses of cognac.
b. \#She added three expensive glasses(ful) of cognac to the sauce ${ }^{3}$

Further, in the individuating constructions, where the numerical expression raises to determiner position, it must be the highest element in the NP. In the measure constructions, since the numerical is part of a measure predicate, it should be able to scope under another modifying adjective. This is also borne out:
(25) a. You drank/spilled an expensive three glasses of wine!
b.\#The waiter brought an expensive three glasses of wine!
c. An expensive ten seconds of silence on the international telephone line followed.
(Sarah Caudwell: Thus was Adonis Murdered )

[^2]Stronger support for the different compositional analyses based on data from Modern Hebrew is presented in Rothstein 2009. There it is shown that the expression in (26) is ambiguous between an individuating and a measure interpretation:
(26) šloša bakbukey yayin
three bottles wine
"three bottles of wine"

Rothstein 2009 shows that (26) is a construct state form, following Borer 1999, 2008. The two nouns babkukey and yayin are part of a construct state, or syntactic word, and this is indicated by the morphological marking on the first noun bakbukey, which has the marked phonologically reduced form, instead of the free absolute plural form bakbukim. In (26), the complement yayin is an NP predicate, and the relation between bakbukey and yayin is not thematically constrained. Rothstein argues that in the construct state, the string [Num $\mathrm{N}_{1} \mathrm{~N}_{2}$ ] can be analysed either with $\mathrm{N}_{1}$ as the head and $\mathrm{N}_{2}$ as the complement as in (27a), giving the individuating reading, or with $\mathrm{N}_{2}$ as the head and Num $+\mathrm{N}_{1}$ modifiying the head as in (27b), giving the measure reading.
(27) a. individuating reading: [ šloša [bakbukey ${ }_{\text {HEAD }}$ yayin $_{\text {COMPLEMENT }}$ ]]
b. measure reading: [[ šloša bakbukey] ${ }_{\text {MODIFIER }}$ yayin HEAD ]

The analysis predicts that if the numerical and the measure expression cannot be combined to form a complex predicate, only the individuating reading should be possible. This is borne out in two constructions, both involving definiteness.

First, definite numerical constructions, unlike the indefinite construction in (26), are necessarily right branching. In both the indefinite NP šloša bakbukim 'three bottles' and the indefinite classifier construction in (26), šloša, 'three' is a simple prenominal adjective. However, all definite numerical NPs must be in the construct state. In (28a), the numerical is morphologically marked for the construct state form šlošet, and heads the simple definite numerical. Definite classifier constructions are illustrated in (28b). In both cases, the definite clitic ha- appears only on the most deeply embedded nominal, but percolates semantically to all elements in the complex nominal.
a. šlošet ha- bakbukim.
three DEF-bottles.
"the three bottles".
b. šlošet bakbukey ha-yayin
three bottles DEF-yayin
"The three bottles of wine"
c. [šlošet [bakbukey ha-yayin] $\left.]_{\mathrm{CS}}\right]_{\mathrm{CS}}$

In the definite classifier nominal (28b), both šlošet 'three' and bakbukey are morphologically marked for the construct state form, and as such must be followed by constituents. Thus the only possible analysis is right branching, as in (28c), with the construct state bakbukey yayin 'bottles of wine' embedded as the syntactic complement of šlošet. As a consequence, bakbukey cannot be construed with šlošet and cannot form a measure modifier, and only the individuating reading is
possible. We can show this as follows: suppose I invite 20 guests and make soup for them, in a big pot. Only seventeen guests arrive, and I say, using a measure construction with a numerical, "The last three bowls of soup were left in the pot". If I try to express this using a definite numerical construct state, šaloš ka'arot ha- marak 'the three bowls of soup' as in (29), the result is infelicitous:
(29) \#šaloš ka'arot ha- marak (ha- axaronot) nišaru b-a- sir. three bowls DEF soup DEF last remained in DEF pot

The only possible reading is the improbable individuating/container reading, where I claim that three bowls, each filled with soup, are still in the pot.

The second prediction is that definite measure constructions are ungrammatical. If a definite construct state nominal does not allow a measure reading syntactically but the content of the construct state only allows a measure reading semantically, then we will get a conflict between syntax and semantics which will result in an ungrammatical construction. This is illustrated in (30). (30a) shows that indefinite measure constructions are possible with measure heads such as kilo, and (30b) shows that the definite forms are not grammatical.

> a. xamiša kilo kemax
> 5 kilo
> "five kilos of flour"
> b. *xamešet kilo ha- kemax
> five kilo DEF- flour
> intended reading: "the five kilos of flour"

We see that we can distinguish grammatically between individuating and measure pseudopartitive constructions. Individuating pseudopartitives have the semantics of complex count nouns, and the numerical is interpreted as a cardinal adjective modifying the nominal head. In measure pseudopartitives, the numerical is a numeral, and it combines with the measure word to form a complex predicate expressing a measure property. These constructions are thus additional contexts in which the numerical expression is a numeral denoting a number at type $n$. ${ }^{4}$

## 3 Numerical classifiers

### 3.1 Numbers as classifiers in English

Numericals have a third use in pseudopartitives as numerical classifiers, as in (31), where they are used to give a very broad estimate of the cardinality of a group, by giving the highest relevant multiplicand.
(31) a. Hundreds of people gathered in the square.
b. She has thousands of books in her library.
c. Hundreds of thousands of people were at the demonstration.

[^3](31a) means something like "the cardinality of the group of people gathered in the square could be counted in hundreds", and so on. The most important generalization about these numeral classifiers is that not all number words have a classifier use. Only multiplicative numerals, i.e. those numericals which must be preceded by another simple numerical, as in two hundred can be used as classifiers. (For clarity, we will call two in this context the 'determiner numerical', or 'determiner'.) Multiplicative numerals include the decimal powers, i.e. hundred, thousand, million, myriad, and a few extra ones including score and dozen. ${ }^{5}$
(32) a. Dozens/scores/hundreds/thousands of people were waiting.
b. Two dozen eggs; Two hundred people.
c. "Four score and seven years ago...." (A. Lincoln)

Numericals which do not combine with another numerical in this way do not have a classifier use, (33) illustrates this for twenty and sixty.
(33) a. Twenty/thirty/sixty cats were in the garden.
b. *Three twenty/two thirty cats were in the garden.
c. *Twenties/thirties/sixties of cats were in the garden.
(34) shows that this is because of constraints on the pseudopartitive construction, and not because pluralizing numbers is impossible.
(34) I arranged the packs in twos/twenties.

Numericals like hundred have different properties depending on whether they occur as names or predicates on the one hand, or as classifiers on the other. As classifiers, the plural marking is obligatory (see ( $35 \mathrm{a} / \mathrm{b}$ ), and the determiner numerical is impossible.
(35) a. *hundred of cats/score of cats
b. *two hundred of cats

Conversely, in non-classifier uses, either as numerals or as cardinal adjectives, the determiner number is obligatory and the plural agreement is impossible.
(36) a. two hundred cats
b. *hundred cats/thousand cats
c. *hundreds cats/thousands cats
d. *two hundreds /three scores cats

This supports the claim made in section 1.1 that multiplicative numerals have a different semantics and syntax from simple number expressions. Clearly, the interpretation of numbers as classifiers exploits this distinction.

[^4]
### 3.2 A tentative semantics

We will follow the semantics suggested in section one. Most numerical expressions are born at type n, but multiplicative numbers, which have also a classifier use, are born at type $<\mathrm{n}, \mathrm{n}>$.
Hundred denotes a function from numbers into numbers which are multiples of a hundred: where the input argument is the number n , the value is a $\mathrm{n} \times 100$.
a. $\llbracket$ hundred $\rrbracket=\lambda \mathrm{n} . \mathrm{n} \times 100$.
b. $\llbracket$ two hundred $\rrbracket=\lambda n . n \times 100[2]=2 \times 100=200$

As a cardinal modifier, two hundred shifts to type <d,t>, denoting the set of plural entities with two hundred non-overlapping atomic parts.
(38) two hundred $=\lambda x .|x|=2 \times 100$

Since hundred /thousand is of type <n, $\mathrm{n}>$, we predict the ungrammaticality of (36b/c). Since two hundred/three score are names for individual numbers at type n , we explain why there is no plural agreement on hundred/score. Presumably in a hundred cats the indefinite determiner induces existential quantification over the $n$ variable. ${ }^{6}$

The meaning of the classifier in the pseudopartitive is hard to pin down. A plausible paraphrase for (31a), hundreds of people gathered in the square is given in (39):
(39) The number of people who gathered in the square is somewhere in the hundreds.

Hundreds of gives us a vague estimate of the cardinality of the pluralities in the denotation of NP. It specifies the largest multiplicative power in terms of which the cardinality of $x$ can be estimated. Note that this is not the round number estimation discussed in Krifka 2009. Krifka is interested in approximative uses of numbers in their non-classifier use, as when four hundred is used to mean about four hundred, with the interpretation in (40):
(40) $|x| \simeq 400$ i.e. the cardinality of $x$ is approximately 400 , or $|x|=400 \pm 15$

I tentatively suggest that classifiers are interpreted as predicate modifiers as in (41b), ${ }^{7}$ crucially using the meaning for hundred at type $<\mathrm{n}, \mathrm{n}>$ given in (37), and repeated here as (41a):
(41) a. 【hundred 】 $=\lambda \mathrm{n} . \mathrm{n} \times 100$ (=37a)
b. $\llbracket$ hundreds $\rrbracket=\lambda P \lambda x . P(x) \wedge|x|>2 \times 100$

[^5]The morphological pluralization operation on hundred indicates that a three stage operation has occurred．First，the number at type $<n, n>$ has been applies to the number 2 ，to give $2 \times 100$ ．This then shifts to the predicate type，but instead of giving the set of entities whose cardinality is $2 \times 100$ ，it gives the set of entities whose cardinality is greater than $2 \times 100 ; \lambda \mathrm{x} .|\mathrm{x}|>2 \times 100$ ． This then shifts to the predicate modifier type，as in（41b）．Hundreds of cats has the interpretation in（42），i．e．it denotes pluralities of cats whose cardinality is over two hundred．
（42）$\lambda P \lambda x . \mathrm{P}(\mathrm{x}) \wedge|\mathrm{x}|>2 \times 100[\lambda \mathrm{x} . \operatorname{CATS}(\mathrm{x})]$
$=\lambda x . \operatorname{CATS}(\mathrm{x}) \wedge|\mathrm{x}|>2 \times 100$
Hundreds of thousands of cats involves predicate composition．We compose the denotation of hundred with the denotation of thousand and then shift it to the classifier meaning to form the complex classifier hundreds of thousands of $N$ ．
（43）a．【 hundred $\rrbracket$ 。 $\llbracket$ thousand 】
$\lambda \mathrm{n} . \mathrm{n} \times 100 \circ \lambda \mathrm{n} . \mathrm{n} \times 1000=\lambda \mathrm{n} . \mathrm{n} \times 100 \times 1000$
b．hundreds of thousands：$\lambda \mathrm{x} .|\mathrm{x}|>2 \times 100 \times 1000$
c．hundreds of thousands of cats：$\quad \lambda \mathrm{x} \cdot \operatorname{CATS}(\mathrm{x}) \wedge|\mathrm{x}|>2 \times 100 \times 1000$

## 3．3 Numbers as classifiers in Modern Hebrew

The contrasts between classifier and cardinal uses of numericals that we saw above also show up in Modern Hebrew，which，because of the peculiarities of the construct state construction， provides explicit support for the analysis of multiplicative numericals given in section 3．2． Remember that a simple cardinal adjective looks like（44）．
（44）šloša bakbukim
three bottles
Multiplicative numerals in Modern Hebrew appear prenominally，like the simple numerals in （44）．The numerical heads $m e ' a$＇hundred＇and elef＇thousand＇appear bare，with the meaning＇one hundred＇，＇one thousand＇，both in counting contexts as numerals and as cardinal adjectives as illustrated in（45）．This is not surprising，since there is no lexical indefinite article in Hebrew，as noted in footnote 6 ．
a．mea xatulim
hundred（SG）cats
＂a hundred cats＂
b．elef xatulim
thousand－SG cats
＂a thousand cats＂

Multiplicative numericals have the forms in（46）．
a．šloš me＇ot three－F hundred－F－PL ＂three hundred＂
b．šlošet alafim three－M thousand－M－PL ＂three thousand＂

As (46) shows, a complex numerical with a multiplicative head is a construct state. Instead of being in the absolute form, šloša (or šaloš for the feminine) as in (44), the determiner numeral is in the reduced phonological form used in the construct state (c.f. (28) above) and agrees in gender with the multiplicative numeral: the feminine form šloš appears with me'a 'hundred' which is feminine, and šlošet, which is masculine, appears with elef 'thousand' which is also masculine.

Complex multiplicative numericals are constrained by general agreement properties concerning numerals. In Modern Hebrew, the plural marking on the head noun is determined by the value of the numerical expression. Low numericals require plural marking on the modified head, high numericals allow singular nouns. This is illustrated in (47):

> (47) a. Šloša anaš-im/ yelad-im three man-PL/child-PL "three men/children"
b. šlošim iš /yeled thirty man-SG/child-SG "thirty men/children"

Complex numerals follow the same principle. In (48a), and (48c), where the determiner numerical is low (three), the multiplicative numerical is in the plural, and in (48b), where the determiner is high (thirty), it is in the singular:
a. šlošet alaf-im three thousand-PL
b. šlošim elef thirty thousand-SG
c. šloš me'ot three hundred-PL

However, while singular/plural agreement in (48) patterns like (47), there is an important difference. In (47), the numerical is in the absolute form. In the complex numerals in (48) the determiner numeral is in the reduced form used in the construct state, indicating that the determiner and the multiplicative numerical are together a 'syntactic word' in the construct state. (This is phonologically explicit in (48a) and (48c), but not in (48b) since there is independent evidence that šlošim 'thirty' has the same form in both the absolute and construct states.)

Complex numericals function as adjectival predicates just like the simple numerical in (44). Predictably, since the cardinal numerical is a multiple of 1000 , and so high, the noun is in the singular: ${ }^{8}$

```
a. elef iš
    thousand-SG people-SG
    "a thousand people"
```

b. šlošet alafim iš three thousand-PL people-SG "three thousand people"

We now compare the behavior of the multiplicative numerical in cardinal and in classifier constructions. ${ }^{9}$ The Hebrew equivalent of the English thousands of people is (50). Crucially, alfey 'thousands' is in the construct state as opposed to the absolute plural form (which shows up in (48a) and (49b)) and the nominal head is plural. The classifier use of alfey with a singular noun is ungrammatical (50b), and as in English, no determiner numerical is possible (50c).

[^6](50)
a. alfey anašim
thousand-CS-PL men-PL
"thousands of people"
b. *alfey iš
thousand-CS-PL men-SG
c. *šlošet alfey anašim three-CS thousand-CS-PL men-PL

The morphological evidence clearly indicates that in the classifier construction, alfey and the N following it form a construct state, or syntactic word, while the complex numerical šlošet alafim forms an independent construct state, which modifies an syntactically independent nominal hea, d outside the construct state. The two contrasting structures are shown in (51). (49b) has the structure in (51a) and (50a) has the structure in (51b):
(51) a.

alfey anašim

The difference in the inflection on alafim and alfey shows that the classifier construction in (51b) is right-branching, while (51a), is not right-branching. Instead, the complex numeral šlošet alafim is a constituent, modifying the nominal head just as the simple numeral modifies the head in (44a). Thus the contrast between elef in its classifier and its cardinal uses is morphologically explicit. A recursive right-branching structure would have to have the morphological form in (52), but this is ungrammatical, with either a singular or a plural nominal.
(52) a. *[šlošet [alefey [iš ]]] three thousand-PL- CSpeople-SG
b. *[šlošet [alefey [anašim]]] three thousand-PL- CS people-PL

The Hebrew data thus supports the analysis in which elef is of type $<\mathrm{n}, \mathrm{n}>$, and denotes $\lambda \mathrm{n} . \mathrm{n} \times$ 1000. This combines with a numeral to give the a complex numeral, expressed in a construct state, e.g. šlošet alafim, denoting $3 \times 1000$. This numeral shifts to the predicate type and modifiers the N as any other number. The classifier alfey, like thousands, is a morphologically marked as a predicate modifier at type $\langle<\mathrm{d}, \mathrm{t}\rangle,<\mathrm{d}, \mathrm{t}\rangle>$, and heads a construct state form.

## 4 Conclusions

We have seen several different uses of numericals. Simple numerals are of type $n$ and multiplicative numericals are of type $<\mathrm{n}, \mathrm{n}>$ and combine with a simple numeral to give complex numerals such as $n$ hundred at type $n$. These are used to denote numbers in counting, in statements about numbers and in measure constructions. Simple and complex numerals can
shift to the predicate type, and function as cardinal modifiers. Multiplicative numerals shift to the predicate modifier type <<d,t>, <d,t>> and occur in pseudopartitives as in hundreds of people

This means that we can distinguish three different kinds of pseudopartitive constructions, as in (53), where in each case the numerical appears in a different form:
(53) a. three kilos of flour
b. three boxes of books
c. hundreds of cats, thousands of people.
(53a) is a measure pseudopartitive, where the numerical is interpreted at type $n$, (53b) has an individuating interpretation (as well as a measure interpretation) in which the numerical is a cardinal adjective, and (53c) is a numerical classifier construction.

## Acknowledgements

This research was supported by Israel Science Foundation Grant 851/10. I presented this work at the $16^{\text {th }}$ Sinn und Bedeutung Conference in Utrecht and at seminars at the University of Düsseldorf and at the Hebrew University of Jerusalem. I thank organisers of all these events for giving me the opportunity to present this material, and the audiences for comments and suggestions, with special thanks to Mark Steiner of the Hebrew University. Fred Landman and Dafna Rothstein Landman were generous with judgments and comments, despite the fact that I asked for them during our vacation. My nephew Ben Rothstein verified the data in (5b). Special thanks also to Angelika Kratzer, who, almost fifteen years ago, gave Dafna an anthology containing Wanda Gag's story Millions of cats about a man who went to look for a cat and came back with "...hundreds of cats, thousands of cats, millions and billions and trillions of cats..." These examples have been bothering me ever since.

## References

Borer, Hagit. 1999. Deconstructing the construct. In Beyond Principles and Parameters, eds. K. Johnson and I. Roberts, Dordrecht: Kluwer publications
Borer, Hagit. 2008. Compounds: the view from Hebrew. In The Oxford Handbook of Compounds, eds. R. Lieber and P. Stekauer, Oxford: Oxford University Press.
Carslon, Greg. 1977. Reference to Kinds in English. Ph.D. dissertation, University of Massachsetts at Amherst,
Chiercha, Gennaro. 1984. Topics in the Syntax and Semantics of Infinitives and Gerunds. Ph.D. dissertation, University of Massachusetts at Amherst.
Chierchia, Gennaro. 1998. Plurality of mass nouns and the notion of 'semantic parameter'. In Events and grammar, ed. S. Rothstein, Dordrecht: Kluwer.
Chierchia, Gennaro and Raymond Turner. 1988. Semantics and Property Theory, Linguistics and Philosophy 11.3: 261-302.
Danon, Gabi. 2011. Two structures for numeral-noun constructions ms. Bar-Ilan University.
Doetjes, Jenny. 1997. Quantifiers and Selection. Ph.D. dissertation, University of Leiden.
Ionin, Tania, and Ora Matushansky. 2006. The composition of complex cardinals. Journal of Semantics 23, 315-360.
Krifka, Manfred. 2009. Approximate interpretations of number words: A case for strategic communication. In E. Hinrichs \& J. Nerbonne (eds.), Theory and Evidence in

Semantics, Stanford: CSLI Publications 2009, 109-132.
Landman, Fred 2003. Predicate-argument mismatches and the adjectival theory of indefinites. In M.Coene and Y. d'Hulst eds. From NP to DP: Volume 1. Amsterdam, John Benjamins.
Landman, Fred. 2004. Indefinites and the Type of Sets. Oxford: Blackwell.
Partee Barbara, and Vladimir Borschev. (in press). Sortal, relational and functional interpretations of nouns and Russian container constructions. Journal of Semantics.
Rothstein, Susan. 2009: Measuring and counting in Modern Hebrew. Brill's Annual of Afroasiatic Languages and Linguistics, Volume 1. 2009. 106-145.
Rothstein, Susan. 2010a. Counting and the mass-count distinction. Journal of Semantics. doi:10.1093/jos/ffq007.
Rothstein Susan, 2010b. Counting, measuring and the semantics of classifiers. The Baltic International Yearbook of Cognition, Logic and Communication Volume 6. http://dx.doi.org/10.4148/biyclc.v6i0. 1582
Rothstein, Susan (in preparation). Some syntactic and semantic properties of construct state expressions in Modern Hebrew. (Working title). To appear in Italian Journal of Linguistics, special issue, edited by Lisa Cheng.
Selkirk, Elisabeth. 1977, Some remarks on Noun Phrase Structure. In Formal Syntax. eds. P.Culicover, T. Wasow and A. Akmajian. London: Academic Press 285-316.


[^0]:    ${ }^{1}$ More properly, the numeral $n$ at type $n$ denotes the individual correlate of the equivalence class, or set of entities with n atomic parts, in the sense of Chierchia 1984, Chierchia and Turner 1988. So two denotes $\cap\{x:|x|=2\}$ and $v^{u n}\{x:|x|=2\}=\{x:|x|=2\}$. I work out the semantics of this explicitly in work in progress.

[^1]:    ${ }^{2}$ For discussion of the relation between the simple nominal glass, the relational nominal glass and the measure head glass see Partee and Borschev, in press.

[^2]:    ${ }^{3}$ An adjective in this position is possible if it modifies the measure unit e.g. Add three heaped teaspoonfuls of sugar.

[^3]:    ${ }^{4}$ Rothstein 2010b shows that measure pseudopartitives have the semantics of mass nous.

[^4]:    ${ }^{5}$ The only exception is tens. We can (marginally) say tens of people. We can certainly say tens of thousands of people but we cannot use *two ten people. Arguably, this is because twenty is derived from two ten.

[^5]:    ${ }^{6}$ Hebrew allows me'a xatulim literally 'hundred cats'. But since there is no indefinite determiner in Hebrew this is not suprising. More suprising is that Dutch allows both een honderd katten and honderd katten. The first means "one hundred cats" and the second is unstressed and means " a hundred cats". Presumably there is a null existential quantifier in the second case.
    ${ }^{7}$ The semantic type of numerical classifiers is thus what Ionin and Matushansky propose as the type for all nominals. I compare my account to their account explicitly in work in progress. See also the discussion of the syntax of nominals in Danon 2011 ms.

[^6]:    ${ }^{8}$ As one of my informants told me, it would be more politically correct to express three thousand people as šlošet alafim iš ve-iša '3000 man and woman'.
    ${ }^{9}$ We will look only at examples with elef since the contrasts show up only when the multiplicative numeral is masculine. Feminine plural forms are the same for both the absolute and construct states.

